Online Supplement for Energy-Efficient Cluster-Head Rotation in Beacon-Enabled IEEE 802.15.4 Networks

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This Online Supplement is organized as follows: Section A briefly describes the operation of the transmission tax-based MAC protocol used for probabilistic rendezvous procedure. Section **??** describes the details of the analytical model for probabilistic rendezvous. Section **??** describes the details of the analytical model for sequence-based rendezvous.

A. Analysis of the algorithm

The performance of the algorithm described above can be modeled using Markov chains and probabilistic analysis. Note that the transition between successive macrocycles occurs with probability of 1, and the same observation holds for the transition between rounds in a macrocycle, and for microcycles within a round. As the result, the resulting DTMC is irreducible, recurrent and apperiodic, which means it is ergodic and has a stationary distribution. We also assume that battery capacity is sufficiently large so that the steady state of DTMC lasts sufficiently long time.

As noted above, all packet transmissions use slotted CSMA-CA as required by the standard [1]. A Markov sub-chain for a single CSMA-CA transmission is shown in Fig. 1 (adapted from [2]). The delay line models the requirement that a transmission which cannot be fully completed within the current superframe has to be delayed to the beginning of the next superframe. The probability that a packet will be delayed is $P_d = \overline{D}_d/SD$, and $\overline{D}_d = 2+\overline{G}_p+1+\overline{G}_a$ denotes the total packet transmission time including two clear channel assessments (CCAs), transmission time $\overline{G}_p$, waiting time for the acknowledgement and acknowledgement transmission time $\overline{G}_a$. (We assume that packet transmission will be repeated until successful.) The block labeled T_r stands for $\overline{D}_d$ consecutive backoff periods needed for actual transmission. Impact of noise from the physical layer is modeled with packet error rate (PER), which depends on the more common bit error rate (BER); the probability of packet being correctly received is $\delta = 1 - PER = (1 - BER)^{\overline{G}_p+\overline{G}_a}$. Probabilities that first CCA, second CCA, and the actual transmission will be successful will be denoted as α, β, and γ, respectively.

Within the transmission sub-chain, the process $\{i, c, k, d\}$ indicates the state of the device at backoff unit boundaries where $i \in \{0 \,..\, m\}$ is the index of the current backoff attempt (m is a MAC-defined constant with the default value of 4). The

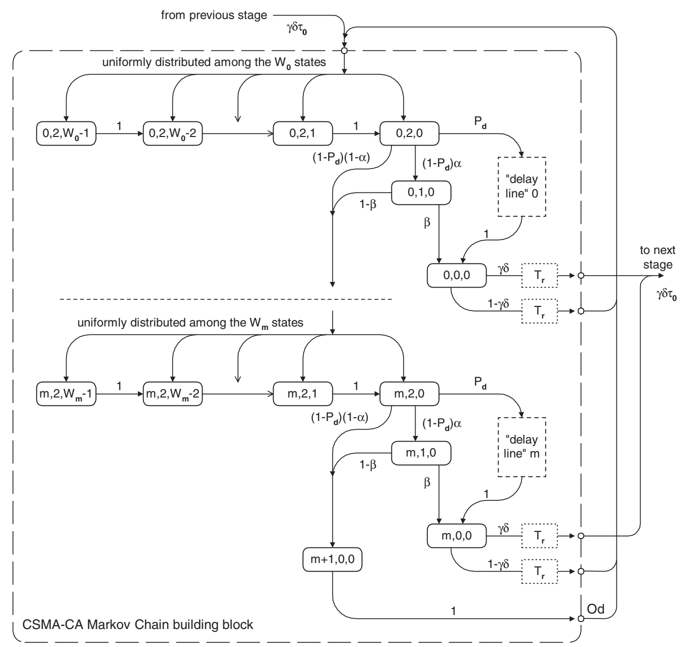

Fig. 1. Markov sub-chain for single CSMA-CA transmission

index of the current CCA phase is $c \in \{0, 1, 2\}$ (two successful CCAs after the backoff countdown are required to begin a transmission). Parameter $k \in \{0 \,..\, W_i - 1\}$ is the value of backoff counter, with $W_i = W_0 2^{min(i,\, 5-macMinBE)}$ being the size of backoff window in i-th backoff attempt while $W_0 = 2^{macMinBE}$ is the minimum window size, with $macMinBE = 3$). Parameter $d \in \left\{0 \,..\, \overline{D}_d - 1\right\}$ denotes the index of the state within the delay line in the case when packet transmission has to be deferred to the next superframe. (For clarity, this will be shown only within the delay line and omitted elsewhere.) Finally, access probability of the general CSMA-CA block will be denoted as τ_0.

When a node wakes up and finds a packet in its buffer, it will wait for (i.e., synchronize with) the beacon before it can begin its backoff countdown. The synchronization time is uniformly distributed between 0 and $BI - 1$ backoff periods, and its Probability Generating Function (PGF) is $D(z) = \frac{1-z^{BI}}{BI(1-z)}$. The sum of probabilities within the beacon synchronization time during steady-state phase is $s_b = \tau_0 \gamma\delta \sum_{i=0}^{BI} \frac{i}{BI}$. Assuming that sleep time is geometrically distributed with parameter P_{sleep}, the sum of probabilities of being in single sleep is $s_{s1} = \frac{\tau_0\gamma\delta}{(1-P_{sleep})}$.

Since the size of the first backoff window is between 0 and 7 backoff periods, $P_d = \frac{1}{8}$ denotes the probability that a packet transmission will occur immediately after the beacon with successful first and second CCAs. Therefore, the probability of finishing the first backoff phase in the transmission block is $x_{0,2,0} = \tau_0\gamma\delta + \tau_0(1-\gamma\delta) = \tau_0$.

Using the transition probabilities indicated in Fig. 1, we can derive the relationships between the state probabilities and solve the Markov chain. For brevity, we omit the detailed derivation; the sum of probabilities for one transmission sub-chain is

$$\begin{aligned} s_t = \;& \tau_0 C_4 \left(C_3(\overline{D}_d - 2) + \alpha(1-P_d) + \frac{P_d(\overline{D}_d - 1)}{2} \right) \\ +\;& \tau_0 \left(\sum_{i=0}^{m} \frac{C_2^i (W_i+1)}{2} + C_2^{m+1} \right) \end{aligned} \tag{1}$$

where $C_2 = (1-P_d)(1-\alpha\beta)$, $C_3 = (1-P_d)\alpha\beta$ and $C_4 = \frac{1-C_2^{m+1}}{1-C_2}$.

Let P_c denote the probability that a node wakes up from sleep to find its buffer empty (and begins a new sleep). The sum of probabilities of being in consecutive sleep periods then becomes $s_s = \frac{s_{s1}}{(1-P_c)}$, and the normalization condition for the whole Markov chain becomes $N_\mu n_c(s_s + s_b + s_t) = 1$.

The total access probability for a particular packet type can be obtained by summing the corresponding access probabilities in a macrocycle:

$$\tau = N_\mu n_c \tau_0 \tag{2}$$

A. The packet queue at a node

We assume that packets arriving to each node follow the Poisson process with a rate of λ_n, and that nodes transmit a single packet before going back to sleep, which corresponds to the so-called 1-limited scheduling [5]. Let $T_t(z)$ denote the PGF for the packet service time, the exact value of which will be derived in Section B below. We need to consider the MAC layer of a node as a $M/G/1/K$ queuing model with a number of Markov points that model the sleep time, synchronization time to the beacon and packet service time. To this end, we need to select Markov points at which we will model the state of the node buffer, as follows.

The PGF for the number of packets that arrive during packet service time is $B(z) = \sum_{k=0}^{\infty} b_k z^k = T_t^*(\lambda_n - z\lambda_n)$, where $T_t^*()$ denotes the Laplace-Stieltjes Transform (LST) [5]. Since service time is a discrete random variable, the LST is obtained by replacing the variable z with e^{-s} in the expression for $T_t(z)$. Let ν_k, $0 \le k < L-1$, denote the probability of having k packets in the buffer after packet departure.

The PGF for a single sleep period is $V(z) = \sum_{k=1}^{\infty}(1-P_{sleep})P_{sleep}^{k-1}z^k = \frac{(1-P_{sleep})z}{1-zP_{sleep}}$. The PGF for the number of packet arrivals during sleep time [5] is $E(z) = \sum_{k=0}^{\infty} e_k z^k = V^*(\lambda_n - z\lambda_n)$. Let ω_k, $0 \le k \le L$, denote the probability of having k packets in the buffer after a sleep.

The PGF for the number of packets that arrive during the synchronization time is $F(z) = \sum_{k=0}^{\infty} f_k z^k = D^*(\lambda_n - z\lambda_n)$. Let δ_k, $0 \le k \le L$, denote the probability of having k packets in the buffer after the synchronization phase.

By modeling the buffer state in Markov points of different types, the steady-state equations for state transitions are

$$\begin{aligned} \omega_0 =\;& \omega_0 e_0 + \nu_0 e_0 \\ \omega_k =\;& \omega_0 e_k + \nu_0 e_k + \sum_{j=1}^{k} \nu_j e_{k-j}, k = 1\,..\,L-1 \\ \omega_L =\;& (\omega_0 + \nu_0)\sum_{k=L}^{\infty} e_k + \sum_{j=1}^{L-1}\nu_j \sum_{k=L-j}^{\infty} e_k \\ \delta_k =\;& \sum_{j=1}^{k} \omega_j f_{k-j}, \qquad k = 1\,..\,L-1 \\ \delta_L =\;& \sum_{j=1}^{k} \omega_j \sum_{k=L-j}^{\infty} f_k \\ \nu_k =\;& \sum_{j=1}^{k+1} \delta_j b_{k-j+1}, \qquad k = 1\,..\,L-2 \\ \nu_{L-1} =\;& \sum_{j=1}^{L} \delta_j \sum_{k=L-j}^{\infty} b_k \\ \sum_{k=0}^{L}\omega_k +& \sum_{k=1}^{L}\delta_k + \sum_{k=0}^{L-1}\nu_k = 1 \end{aligned} \tag{3}$$

The probability distribution of the device queue length at Markov points can be found by solving the system (3). The probability that a Markov point corresponds to a return from the vacation and the queue is empty at that moment is $P_c = \omega_0 / \sum_{i=0}^{L}\omega_i$.

Finally, the average value of total inactive time of a node, including possible multiple consecutive sleeps, is $\overline{I} = \frac{1}{(1-P_{sleep})(1-P_c)}$.

B. Transmission success probabilities

There are N_c clusters containing n_c ordinary sensor nodes, with the packet arrival rate of λ_n per node. Therefore, each cluster has (on the average) $n_c = \frac{n}{N_c}$ nodes. Assuming that the arrival process of medium access event follows a Poisson process, we focus on a single target node and model aggregate packet arrival rates of the remaining $n_c - 1$ nodes in the cluster as background traffic. This approximation is valid if the cluster operates below saturation, i.e., the required event sensing reliability per cluster, $R_{cluster} = \frac{R}{N_c}$, is not too high.

Transmission of packets during non-CH states commences within eight backoff periods starting from third backoff period after the beacon. The background packet arrival rate is $\lambda = (n_c - 1)\tau\frac{SD}{8}$.

Packets will experience success probabilities of

$$\begin{aligned} \alpha =\;& \frac{1}{8}\sum_{i=1}^{7} e^{-i\lambda} \\ \beta =\;& e^{-\lambda} \\ \gamma =\;& \beta^{\overline{D}_d} \end{aligned} \tag{4}$$

The first CCA may fail because a packet transmission from another node is in progress. The second CCA, however, will fail only if some other node has just started its transmission, in which case our packet's first CCA coincided with the second CCA of that other packet.

Then, access probability for non-CH nodes can be found as τ from (2), from which we can calculate α, β and γ using (4). For Ch node transmissions, access probability is $\tau_{CH} = n_c \tau$, and the success probability is $\gamma_{CH} = (1 - \tau_{CH})^{\overline{D}_d(N_c-1)}$, since CH competes with all other CHs for access to the BS.

B. Analyzing node lifetime

Let the PGF of the packet length be $G_d(z) = z^d$ where d is a constant representing the length of the packet in backoff periods. Let $G_a(z) = z$ stand for the PGF of the ACK packet duration. Let the PGF of the time interval between the data and subsequent ACK packet be $t_{ack}(z) = z^2$; and PGF for packet transmission time and receipt of acknowledgement as $T_d(z) = G_d(z) t_{ack}(z) G_a(z)$. Then, the PGF for the time needed for one complete transmission attempt, including backoffs, becomes

$$\mathcal{A}(z) = \frac{\sum_{i=0}^{m} \left(\prod_{j=0}^{i} B_j(z) \right) (1-\alpha\beta)^i z^{2(i+1)} \alpha\beta T_d(z)}{\alpha\beta \sum_{i=0}^{m} (1-\alpha\beta)^i} \tag{5}$$

where $B_j(z) = z^{W_j} - 1/W_j(z-1)$ is the PGF for duration of j-th backoff time prior to transmission. The LSTs for energy consumption during wait and reception of acknowledgment, two CCAs and packet transmission are $e^{-s3\omega_r}$, $e^{-s2\omega_r}$ and $e^{-sd\omega_t}$, respectively. The LST for energy consumption while receiving the beacon is $e^{-s3\omega_r}$, as three backoff periods are sufficient for transmitting information about the number of live nodes and requested event sensing reliability. The LST for energy consumption during transmission of the data packet and reception of acknowledgement is $T_d^*(s) = e^{-sd\omega_t} e^{-s3\omega_r}$. The LST for energy consumption for a single transmission attempt then becomes

$$\mathcal{E}_{\mathcal{A}}^*(s) = \frac{\sum_{i=0}^{m} \left(\prod_{j=0}^{i} E_{B_j}^*(s) \right) (1-\alpha\beta)^i e^{-s2\omega_r(i+1)} \alpha\beta T_d^*(s)}{\alpha\beta \sum_{i=0}^{m} (1-\alpha\beta)^i} \tag{6}$$

By taking packet collisions into account [3], the PGF of probability distribution of the total packet service time becomes

$$\begin{aligned} T(z) &= \sum_{k=0}^{\infty} (\mathcal{A}(z)(1-\gamma\delta))^k \mathcal{A}(z)\gamma\delta \\ &= \frac{\gamma\delta\mathcal{A}(z)}{1-\mathcal{A}(z)+\gamma\delta\mathcal{A}(z)} \end{aligned} \tag{7}$$

while the LST for the energy used to service a packet is

$$E_T^*(s) = \frac{\gamma\delta\mathcal{E}_{\mathcal{A}}^*(s)}{1-\mathcal{E}_{\mathcal{A}}^*(s)+\gamma\delta\mathcal{E}_{\mathcal{A}}^*(s)} \tag{8}$$

Finally, average value of energy consumed for packet transmission is $\overline{E}_T = -\frac{d}{ds} E_T^*(s)|_{s=0}$.

a) Energy consumption during initial set-up: During initial set-up, CH nodes send packets in the advertisement, channel request, channel assignment (uplink) and channel declaration sub-phases; they receive packets during membership and channel assignment (downlink) sub-phases. Non-CH nodes send packets during membership sub-phase only, and receive data during membership and channel declaration sub-phases. Therefore, the PGF for the duration of initial set-up phase ($T_{is}(z)$) is product of the PGFs for packet transmission time during advertisement ($T_{ad}(z)$), membership ($T_{mp}(z)$), channel request ($T_{cr}(z)$), downlink request ($T_{dr}(z)$), downlink ($T_d(z)$) and channel declaration ($T_{cd}(z)$) sub-phases, and the LST of energy consumption during initial set-up phase is

$$\begin{aligned} E_{s,C}^*(s) = \; & E_{Tad}^*(s) E_{Tcr}^*(s) E_{Tdr}^*(s) E_{Tcd}^*(s) \times \\ & T_{mp}(e^{-s\omega_r}) T_d(e^{-s\omega_r}) \\ E_{s,nC}^*(s) = \; & E_{Tmp}^*(s) T_{ad}(e^{-s\omega_r}) T_{cr}(e^{-s\omega_r}) \times \\ & T_{dr}(e^{-s\omega_r}) T_d(e^{-s\omega_r}) T_{cd}(e^{-s\omega_r}) \end{aligned} \tag{9}$$

for CH and non-CH nodes, respectively. In the above relations, values of success probabilities (α, β and γ) may be obtained using existing data obtained for the network in non-saturation regime [4].

b) Energy consumption in steady state: In steady state, each round is composed of N_μ microcycles, each of which is composed of three steps: sleep, beacon synchronization and CSMA-CA uplink data transmission. However, all CH nodes are awake during a microcycle, and their average energy consumption is

$$\begin{aligned} E_{m,C}^*(s) &= I(e^{-s\omega_r}) D(e^{-s\omega_r}) e^{-s3\omega_r} T(e^{-s\omega_r}) \\ E_{m,nC}^*(s) &= I(e^{-s\omega_s}) D(e^{-s\omega_r}) e^{-s3\omega_r} E_T^*(s) \end{aligned} \tag{10}$$

for CH and non-CH nodes, respectively. The corresponding LST is

$$\begin{aligned} E_{r,B}^*(s) &= (E_{m,B}^*(s))^{N_\mu} \\ E_{r,nB}^*(s) &= (E_{m,nB}^*(s))^{N_\mu} \end{aligned} \tag{11}$$

for CH and non-CH nodes, respectively.

During a macrocycle which is composed of n_c rounds, each node has to act as CH in exactly one round. Therefore, the LST for energy consumed during a macrocycle is

$$E_M^*(s) = E_{r,B}^*(s) (E_{r,nB}^*(s))^{n_c - 1} \tag{12}$$

If the battery budget is E_{bat} Joules, the average number of macrocycles during lifetime of a node is $\frac{E_{bat}}{\overline{E}_M}$, where $\overline{E}_M$ is the average energy consumed during a macrocycle, and the lifetime of the network is $\overline{L} = \overline{T}_M \frac{E_{bat}}{\overline{E}_M}$, where $\overline{T}_M$ is average duration of a macrocycle.

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